Fraction Addition & Subtraction

Question: Why is the answer to 1/2 + 1/3 not 2/5?

Possible answers to the question are:

1. Are you sure that the answer is not 2/5? Seems sensible that 2/5 is the answer because 1 + 1 is 2 and 2 + 3 is 5. This is just like adding whole numbers.
2. 2/5 is smaller than 1/2. If you already have 1/2 a pie and add 1/3 of a pie to that, you should end up with more than half of a pie. Because 2/5 is smaller than 1/2, 2/5 can't be the answer.
3. When you change the fractions to decimals, the addition becomes .50 + .33. This sum is .83. If you change 2/5 to a decimal you get .4. Because .4 is not the same as .83, 2/5 cannot be the answer.
4. Mark the fractions on a number line and pretend you are going for a walk. The walk starts at 1/2 and you are supposed to walk 1/3 more. You would end up somewhere to the right of 1/2. But 2/5 is to the left of 1/2. That's why it can't be the answer to the sum.

Response 1 is not appropriate. It assumes that fractions are just like whole numbers and therefore the arithmetic should be the same as that for whole numbers.

Responses 2 and 4 are appropriate. They are based on the knowledge that 2/5 is less than 1/2. The explanations involving that knowledge use different metaphors/contexts.

Response 3 is an appropriate procedural response that involves changing notation systems (fraction to decimal).

Note:

Only addition is discussed here. Subtraction would concern the same models but the opposite action (e.g. removal instead of combining) would be involved.

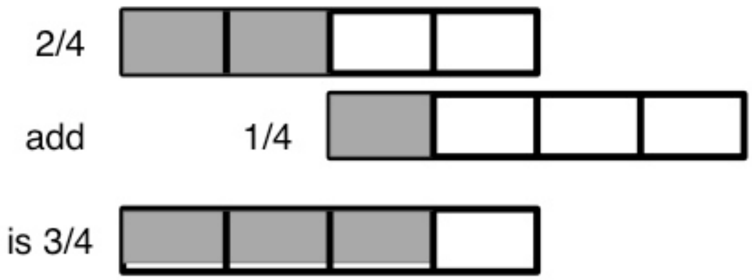
No matter which model is used, it is wise to begin by involving fractions that have the same denominator and then progress to fractions having different denominators.

Models for teaching integer addition/subtraction

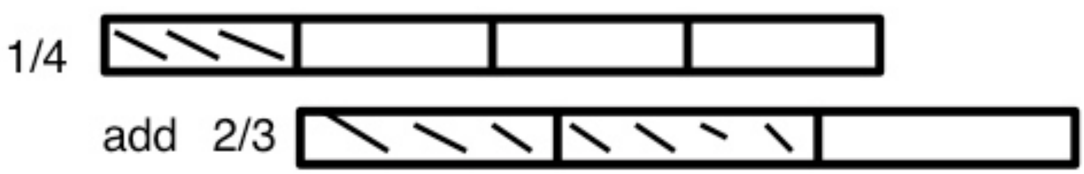
A variety of teaching models can be used to develop fraction addition/subtraction. Note that the part of a set/group meaning of fraction (see: Five meanings of fraction) should not be used. The reasons are: (1) does not involve equality of parts, (2) model looks like whole number addition, and (3) normally is difficult to identify the set involved. The most effective models are the following:

Fraction bar model

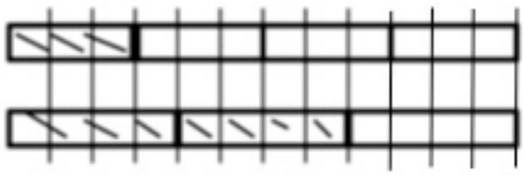
This model involves the part of a whole meaning of fraction. It concerns using a bar subdivided into equal parts to represent fractions. Some refer to this as an area/ region model. An example with same denominators follows for 2/4 add 1/4.



When denominators are different more difficult thinking is involved. Suppose the question is 1/4 add 2/3. The diagram shows that the answer is less than 1 (because the part added to 1/4 does not fill up the entire fraction bar that shows 1/4).



You should understand that the answer is less than 1 (for 1/4 + 2/3) but you also need to determine the exact answer. To do this, both fraction bars must be cut up into the same number of equal parts. We call that number the ‘lowest common denominator (LCD)’. By drawing the fraction bars on grid paper, you should see that 12 works. You can also get this ”magic” number by making a skip-counting list for 4 and for 3.



On grid paper, line up the two fraction bars. Then cut up each section into squares so that the size of each square is the same for each fraction bar. The only way this works is if quarters are cut into 12ths and thirds into 12ths.

For 4: 4, 8, **12**, 16, . . .

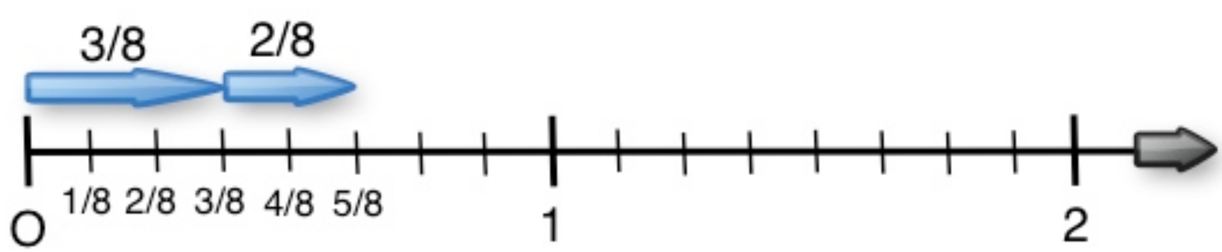
For 3: 3, 6, 9, **12**, 15, . . .

The number 12 is the first number that is common to both lists. It is the LCD and also is known as the lowest common multiple (LCM) of 3 and 4.

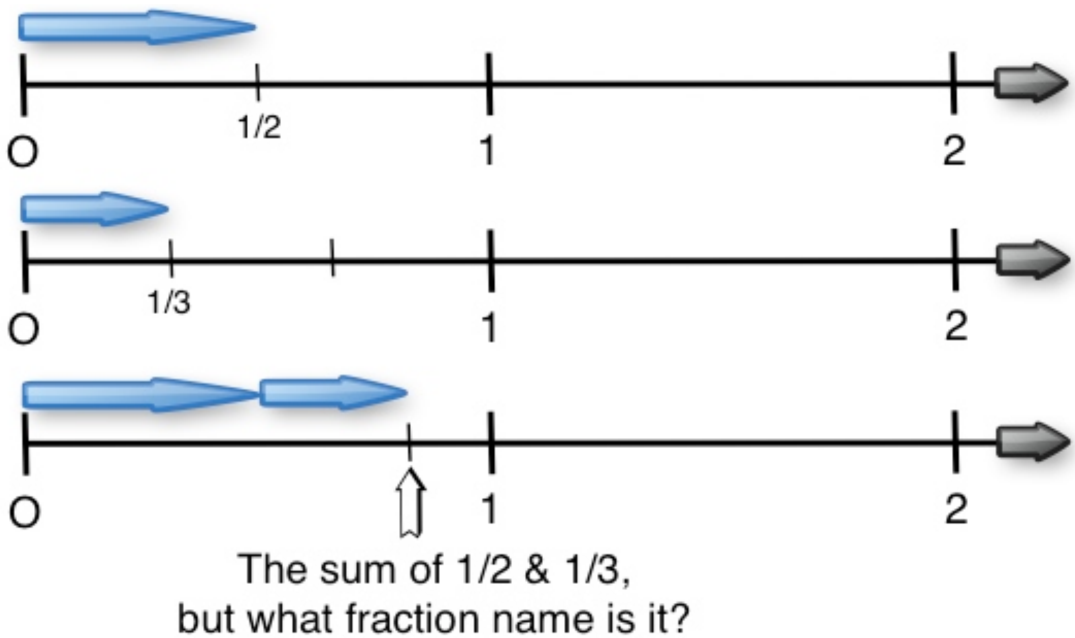
You should see from the fraction bar that 2/3 is the same as 8/12 and that 1/4 is the same as 3/12. The answer, then, to 1/4 + 2/3 is 3/12 + 8/12 = 11/12.

Number line model

This model involves the measure (name for a point on a number line) meaning of fraction. It concerns using a number line with the units subdivided into equal lengths.



An example with same denominators follows for 3/8 add 2/8.
Note that the advantage of using a number line is that the answer can be identified by indicating the fraction name for each subdivision mark. This works well as long as denominators are the same. If they are not the same, the same kind of thinking is involved as for fraction bars. Here is an example for 1/2 + 1/3.

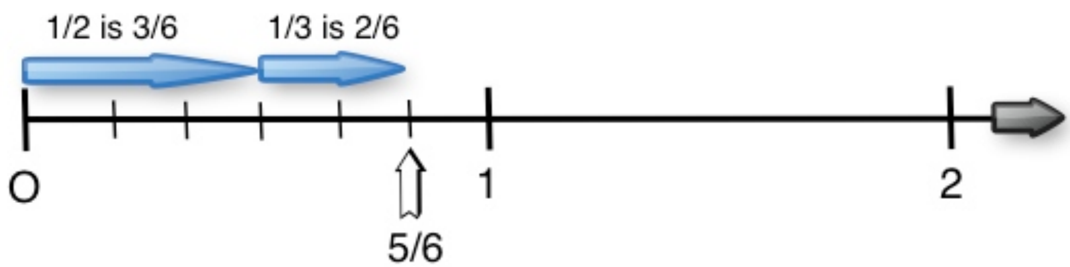


You should realize that the answer for 1/2 + 1/3 is less than 1. Once again, a simple way to determine the LCD is to use skip counting lists.

For 2: 2, 4, **6**, 8, . . .
For 3: 3, **6**, 9, 12, . . .

The number 6 is the first number that is common to both lists. It is the LCD and also is known as the lowest common multiple (LCM) of 2 and 3.

The number line can then be cut into 6th. You should realize that 1/2 and 1/3 can each be cut into sixths exactly.

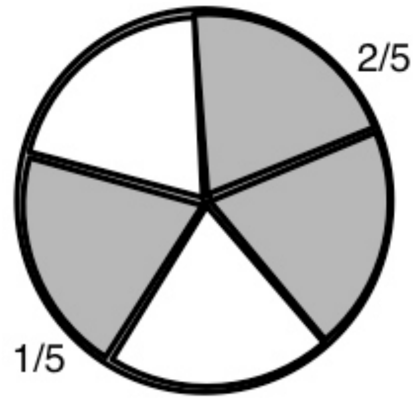


The answer, then, to 1/2 + 1/3 is 3/6 + 2/6 = 5/6.

Pie model

This model involves the part of a whole meaning of fraction. It concerns using a circle subdivided into equal parts to represent fractions. Some refer to this as an area/region model. This model is quite similar to the fraction bar model. The only example shown here is for 2/5 + 1/5 = 3/5.

The technique used for fraction bars applies when adding fractions with different denominators.



PRIME FACTORS (POT) algorithm - EFFICIENT AND ELEGANT for determining the LCD

Clearly using models to determine answers to fraction addition or subtraction is a starting point, not an end point. The models are intended to help explain why common denominators are needed for adding/subtracting fractions and to ensure that the answers you get by using models are understood and believed. At some point, symbolic methods must be developed.

Making skip counting lists is a useful symbolic method but it is inefficient. However it can be used as a stepping stone to a highly efficient and elegant method (which I refer to as the **PRIME FACTORS** algorithm).

Suppose two question is 3/4 + 3/10. The skip-counting method can be used to determine that 20 is the LCD. The two lists are:

For 4: 4, 8, 12, 16, **20**, 24, . . .
For 10: 10, **20**, 30, . . .

Write 20 as product of primes (a factor tree can be used for this).

You should obtain 20 = 2 x 2 x 5.

Write 4 and 10 as a product of primes.

You should obtain: 4 = 2 x 2 and 10 = 2 x 5.

Find 4 as 2 x 2 in 2 x 2 x 5 (20) and find 10 as 2 x 5 in 2 x 2 x 5 (20).

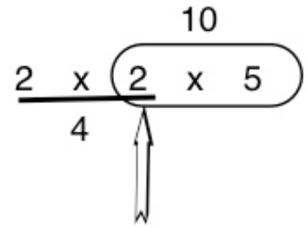
Because you can find 4 and 10 in 2 x 2 x 5, this means that 4 divides into the LCD and 10 divides into the LCD. Each of the denominators must divide exactly into the LCD; otherwise you have not found the LCD. In other words, the prime factors of each denominator must be present in the list of prime factors of the LCD.

Write the product of 4 x 10 as a product of primes.

You should obtain: 4 x 10 is 2 x 2 x 2 x 2 x 5.

Compare [2 x 2 x 2 x 2 x 5] to [2 x 2 x 5].

Do you realize that there is no point in duplicating one of the 2s in the LCD because 2 x 2 x 5 (the LCD) contains 4 (2 x 2) and also contains 10 (2 x 5)? If not, the diagram on the right should help.



One of the 2s does double duty for 20 as the LCD. It is part of 4 and part of 10.

Why **PRIME FACTORS** algorithm? Because it involves writing the prime factors of all denominators and using them to determine the LCD.

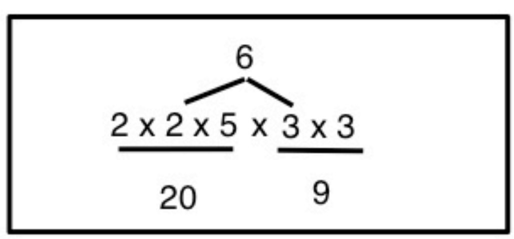
Here is an example for 1/6 + 1/9 + 1/20.

Prime factors of the denominators:

6 = 2 x 3
9 = 3 x 3
20 = 2 x 2 x 5

It makes good sense to place the prime factors of the largest denominator into the POT first. Then put in the prime factors of the remaining denominators one by one, making sure to avoid duplication of not-needed factors. For example, after putting in the factors of 20 and of 9, the factors of 6 (2 x 3), are already in the POT. Therefore, you do not put in another 2 x 3 for 6.

The LCD for 6, 9, and 20 works out to be: 2 x 2 x 5 x 3 x 3 = 180. Confirm that this is the smallest number that each of 6, 9, and 20 divide into.



Refer to: Grade 7 Fraction add & subtract (7.N.5) **if more help is needed.**